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FINAL REPORT

SATURN V-LAUNCHER-UMBILICAL TOWER VIBRATION ANALYSIS

Vol. II

A COMPUTER PROGRAM FOR ANALYSIS OF THE VIBRATIONAL CHARACTERISTICS OF LARGE LINEAR SPACE FRAMES

Lockheed

MISSILES & SPACE COMPANY

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A COMPUTER PROGRAM FOR ANALYSIS OF THE VIBRATIONAL CHARACTERISTICS OF LARGE LINEAR SPACE FRAMES

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FOREWORD

The work described in this report was performed by Lockheed Missiles & Space Company, Huntsville Research & Engineering Center, for the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration under Contract NAS8-21301.

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The final report for "Saturn V-Launcher-Umbilical Tower Vibration Analysis" consists of two volumes:

Vol I: Response to Ground Wind Excitation

Vol II: A Computer Program for Analysis of the Vibrational Characteristics of Large Linear Space Frames

SUMMARY

This report describes a computer program for calculating the modes and frequencies of large undamped linear space frames. The program provides an economical means of accurately computing the vibrational characteristics of complicated frame structures such as the Saturn V launcher-umbilical tower. The computational procedure used in the program deals exclusively with non-zero submatrices of the system mass and stiffness matrices, thus avoiding both trivial arithmetic and wasted data storage space. Accordingly, very low computer execution costs are attained. The solution technique is, in effect, a generalization of the well-known Stodola method of beam vibration analysis.

The program is automatic in the sense that both input and output communications are concise. Input data consist of minimum definitions of particular problems (e.g., joint position coordinates, member properties, restraint conditions, etc.). Output data consists of optional solution information, including computer-generated plots of mode shapes. Several examples are presented of solutions computed by the program.

A wide variety of member types is allowed, including open and non-symmetrical sections. Frame members may be either uniform beams or Timoshenko beams with arbitrarily varying mass and stiffness properties. Provisions are also included for rigid links, lumped masses and other features useful in mathematically modeling complicated linear structures.

The method upon which the program is based is applicable to more general classes of finite-element structures.

CONTENTS

NOMENCLATURE

c_i^j = coefficients of system modes;

e = scalar defined in Eq. 16;

F = vector of generalized forces (static);

K = stiffness matrix of the entire structure;

K_k = stiffness matrix associated with the k-th beam element;

ℓ = length of a uniform beam element;

M = mass matrix of the entire structure;

M_k = mass matrix associated with the k-th beam element;

$\bar{M}_k^{ij}$ = submatrices of the k-th element mass matrix, defined by Eq. 22;

m_k = a scalar defined by Eq. 33;

n = the total number of beam elements contained in the structure;

Q_k^i = transformation matrix defined by Eq. 27;

R_k^i = 3x3 matrix of direction cosines specifying the orientation of the principal axes of the k-th element, relative to a reference frame associated with the i-th node.

T = kinetic energy of the entire structure;

T_k = kinetic energy associated with the k-th element;

t = time;

U = vector of system generalized coordinates (node point motion components);

$\bar{u}^i$ = six-vector containing the motion components of the i-th node, relative to a reference frame associated with an element connected to the node;

V = potential energy of the entire structure;

X_k = k-th system mode;

x^i = a six-vector containing the motion components (associated with a particular eigenvector) of the i-th node, relative to a reference frame uniquely associated with the node;

$\bar{x}^i$ = a six-vector containing the motion components (associated with a particular eigenvector) of the i-th node, relative to a reference frame associated with an element connected to the node;

Y^j = j-th approximation of a system mode;

y^i = a vector analogous to x^i, associated with an approximation of a system mode;

$\bar{y}^i$ = a vector analogous to $\bar{x}^i$, associated with an approximation to a system mode;

μ = mass per unit length of a uniform beam element;

ω = frequency approximation; and

ω_i = frequency associated with X_i.

Section 1
INTRODUCTION

Calculation of the undamped vibrational modes of linear finite element networks requires solution of matrix eigenproblems of the type ω^2 MX = KX. Most digital programs that have been developed for this purpose, including those based on variable-width band matrix procedures, fail to take full advantage of the sparsity of M and K. Such programs consequently are limited to problems of small or moderate size.

The computer program discussed in this report is applicable to a general class of large complex space frames with thousands of degrees of freedom. The method upon which it is based is applicable to any linear finite-element network. The basic computational routine is the digital program described in Reference 1 for analyzing large statically loaded space frames. This program employs a computational procedure which deals exclusively with non-zero submatrices of the stiffness matrix, thus avoiding both trivial arithmetic and wasted data storage space. The "bookkeeping" procedures executed by this program to enable avoidance of trivial operations consume only a negligibly small fraction of the total computer execution time; accordingly, as discussed in detail in Reference 1, very low computer costs are attained.

The solution technique used in the frame dynamics program is, in effect, a generalization of the well-known Stodola method of beam analysis. Beginning with an initial approximation of a system mode (computed by the program on the basis of a static loading condition specified in the input data), an "equivalent inertial loading" acting over the entire structure is evaluated. The static deformation corresponding to the equivalent inertia loading is computed, providing an improved approximation of the mode. This procedure is executed repeatedly until convergence is attained. In computing higher modes, a process based on orthogonality relations is used to "sweep out" previously computed lower modes. This solution technique is, of course, equivalent in principle to the matrix iteration method (see Reference 2), which is one of the oldest eigenproblem solution techniques; the distinguishing feature of the new technique is the avoidance of trivial computation and data storage, which makes possible economical and accurate analysis of extremely large systems. During recent studies of program performance, several lower modes of structures having over 1000 degrees of freedom have been computed using just a few minutes of IBM 7094 time. The program has no explicit degree-of-freedom limitation, and in its present configuration can be used to analyze systems having many thousands of elements.

Several features have been incorporated into the program primarily to facilitate mathematical modeling of a wide variety of structures. These include:

- rigid links offsetting member end points from joints,
- members (may be zero-length) described by 6 x 6 stiffness matrices, and
- additional lumped masses at the joints.

As restraint conditions an arbitrary set of joint motion components may be set indentically equal to zero.

Examples are presented of solutions computed by the program.

Section 2
ITERATIVE PROCEDURE

An arbitrary n degree-of-freedom linear finite element system will be considered. Where U is the vector of generalized coordinates (node point motion components, etc.), M the mass matrix, and K the stiffness matrix, the kinetic and potential energies of the system are

$$T = \frac{1}{2} \dot{U}^* M \dot{U} \text{ , and } V = \frac{1}{2} U^* K U \text{ .} \tag{1}$$

For undamped free vibration, Lagrange's equation gives

$$M \ddot{U} + K U = 0 \tag{2}$$

Solutions of the form $U = \sin \omega t \; X$ yield the usual linear, small-vibration eigenproblem,

$$\omega^2 MX - KX = 0 \tag{3}$$

It will be assumed in the following discussion that the modes, X_1, X_2, ... X_{k-1} associated with the lowest k-1 frequencies have already been calculated, and that it is desired to compute the k-th mode, X_k, associated with the next higher frequency, ω_k. Suppose a function, Y^1, is known which is a linear combination of the unknown modes of the system, X_k, X_{k+1}, ... X_n (an n degree-of-freedom system); that is,

$$Y^1 = \sum_{i=k}^{n} c_i^1 X_i \text{ .} \tag{4}$$

Determination of Y^1 will be discussed subsequently. A sequence of functions, Y^2, Y^3, ..., is computed, subject to the requirement that

$$\omega^2 M Y^j = K Y^{j+1} . \tag{5}$$

Y^j may be written as

$$Y^j = \sum_{i=1}^{n} c_i^j X_i . \tag{6}$$

Substitution of Eq. 6 into 5 gives

$$\omega^2 M \sum_{i=1}^{n} c_i^j X_i = K \sum_{i=1}^{n} c_i^{j+1} X_i . \tag{7}$$

Since $X_r^* M X_i$ and $X_r^* K X_i$ are identically equal to zero for i unequal to r, pre-multiplication of both sides of Eq. 7 by X_r^* gives, for i = 1 through n,

$$\omega^2 X_i^* M X_i c_i^j = X_i^* K X_i c_i^{j+1} . \tag{8}$$

Since

$$\omega_i^2 = \frac{X_i^* K X_i}{X_i^* M X_i} , \tag{9}$$

Eq. 8 may be re-written as

$$c_i^{j+1} = \frac{\omega^2}{\omega_i^2} c_i^j , \tag{10}$$

for i = 1 through n. Note that for i = 1 through k-1, $c_i^j = 0$ for all j, since $c_i^1 = 0$. Since ω must be greater than or equal to ω_k, it is apparent from Eq. 10 that unless $c_k^1 = 0$, the sequence Y^1, Y^2, ... will converge to X_k, and ω will converge to ω_k.

Determination of Y^{j+1} according to the requirements of Eq. 5 may be regarded as a problem in static deformation analysis: Given a vector of applied generalized forces, $F = \omega^2 M Y^j$, determine the corresponding set of generalized displacements, $U = Y^{j+1}$, such that

$$F = K U . \tag{11}$$

Calculation of Y^1 subject to the requirements of Eq. 4 will now be considered. Suppose an initial approximation, Y^o, of the k-th system mode has been computed. Y^o may be expanded in a finite series of the normal modes of the system:

$$Y^o = \sum_{i=1}^{n} c_i^o X_i \tag{12}$$

The requirements of Eq. 4 are satisfied if we choose

$$Y^1 = Y^o - \sum_{i=1}^{k-1} c_i^o X_i \tag{13}$$

Pre-multiplying both sides of Eq. 12 by $X_i^* M$, (or $X_i^* K$) yields, for i=1,2,...k-1,

$$c_i^o = \frac{X_i^* M Y^o}{X_i^* M X_i} = \frac{X_i^* K Y^o}{X_i^* K X_i} \tag{14}$$

Given $X_1, \ldots X_{k-1}$ and Y^o, Eqs. 14 and 13 provide a basis for constructing Y^1.

Section 3
COMPUTER PROGRAM

A digital program based on the iterative procedure outlined above has been developed for computing the modes and frequencies of large arbitrary space frames. The program is coded entirely in FORTRAN-4 and can be executed on any large-scale digital system (IBM 7094, Univac 1107-8, IBM 360-50, CDC 3000 or 6000 series, etc.), having a Fortran compiler and six secondary data storage units (tapes, drums, etc.).

Dynamic allocation of core storage is employed with arrays packed sequentially to avoid the waste of core storage space associated with the use of arbitrarily fixed array dimension specifications. The program can be converted to execute on various computers by changing a few statements in the main program which specify (1) total amount of core space to be used and (2) the secondary data file assignments (unit numbers of tapes, drum areas, etc.). For analysis of small problems (i.e., structures containing a few hundred joints, or less) the program can be executed using less than 15000 core locations, including instruction storage.

For purposes of minimizing input data requirements, frames are represented as arrays of "joints" interconnected by straight "members." Each member is generally subdivided by the program into a specified number of uniform beam elements of equal length, so that there are two classes of nodes: (1) the joints, which usually interconnect several non-colinear members, and (2) nodes interconnecting pairs of colinear beam elements within a member. Input data includes the position coordinates of the joints, restraint conditions, and, for each member, (1) indices identifying the pair of joints it interconnects, (2) information specifying section physical and geometrical properties and cross-section orientation, and (3) the number of beam elements into which it is to be subdivided. Various classes of members are allowed, including several types prismatic open sections (wide-flanges, angles, channels, tees, etc.), Timoshenko beams with section properties given as piecewise linear functions of position along the member's central axis, and sections for which principal moments of inertia, cross-sectional area, mass density, etc., are directly specified. Provision is also included for rigid masses lumped at joints, and for massless rigid links offsetting member end points from joints.

Care has been taken throughout the program to avoid trivial arithmetic operations and to minimize secondary data storage requirements. Transmissions to and from secondary storage units are made via large physical records (typically several thousand words each, for large structures) to reduce access time.

Problem size limitations are determined primarily by the capacity of the static analysis routines used to compute the sequences of functions, Y^j, described in the discussion of the iterative solution procedure. The capacity of the static analysis routines depends upon the amount of core storage space available. A typical IBM 7094 version of the program using a total of approximately 24000 core locations allows up to 1000 joints, with no limit on the number of members, and no limit on the number of colinear beam elements into which the members are subdivided. The static analysis routines used in the program, which are essentially the same as those described in Reference 1, are not based on band matrix methods, hence have no bandwidth restrictions. These routines execute faster (in many cases very substantially faster) than programs based on band matrix methods, unless there are no zeros within the banded portion of the stiffness matrix, in which case execution times are about the same as those obtained using band matrix methods.

3.1 PROGRAM ORGANIZATION

The sequence of operations performed by the program is as follows: Input data is read defining the geometrical and physical properties of the structure, and two data files, which are used repeatedly in the iterative solution process, are generated. One of the files contains compact data arrays which serve essentially the same function as a system flexibility matrix. Details of the composition of these arrays and how they are generated are explained in Reference 1. The other file generated at this time contains, for each member, the following information:

1. Two indices identifying the pair of joints interconnected by the member.
2. The length of the member, and the number of beam elements (of equal length) into which the beam is to be subdivided.
3. 3x3 matrices of direction cosines computed by the program which specify the orientation of the member's principal axes relative to the reference frames to which the motion components of the interconnected joints are referred.
4. The moments of inertia, cross-sectional area, mass density, etc., of each beam element into which the member is subdivided.

After printing a complete description of the structure, the program executes the following steps to compute each required mode.

1. Input data is read defining a static loading which the program uses to compute Y^o, the initial approximation of the mode. These static loadings usually consist of a few point loads applied at joints. Obviously, convergence of the iterative procedure is hastened if care is taken, where possible, to choose a static loading which will produce a displacement field roughly similar to the mode.
2. Y^1 is computed as indicated by Eqs. 13 and 14. The procedure for computing terms of the type X^*MY and X^*KY is discussed subsequently.
3. For j = 1,2,...., the sequence of improved approximations, Y^j, of the mode are computed as indicated by Eq. 11. After evaluation of each Y^j, the sweep-out process indicated by Eqs. 13 and 14 is performed to offset arithmetic error accumulated in the static solution process. The frequency approximation, ω^{j+1}, associated with Y^{j+1} is evaluated as

$$(\omega^{j+1})^2 = \frac{(Y^{j+1})^* K \, Y^{j+1}}{(Y^{j+1})^* M \, Y^{j+1}} \; . \tag{15}$$

Where e is a specified small number (typically 10^{-5}), termination of the iterative process is based on the convergence criterion,

$$\frac{(\omega^{j+1})^2 - (\omega^j)^2}{(\omega^j)^2} \leqslant e\ . \tag{16}$$

Calculation of the equivalent static loading, F, static solutions, and $(Y^{j+1})^* K (Y^{j+1})$ will be discussed subsequently.

4. After completion of the iterative process, solution details are printed and plots are generated of the structural deformation corresponding to the newly-computed mode, X_k. The generalized mass and stiffness, $X_k^* M X_k$ and $X_k^* K X_k$, evaluated during the final step in the iterative process are retained for use in subsequent sweep-out procedures executed during evaluations of higher modes.

3.2 DATA FILES

During execution of the iterative procedure for computing X_1, X_2, ... , two additional data files are generated. One of these contains the most recently-generated approximation of the mode currently being computed. Initially Y^o is stored in this file; then Y^o is replaced by Y^1 which is in turn replaced by Y^2, etc. The other file contains all of the previously-computed modes, X_1, X_2, ... , in serial order. For a particular mode, or modal approximation, these files contain, for each member, the motion components (relative to "element reference frames" discussed subsequently) of each node point in the member. As illustrated on Figure 1, data in these files are arranged to correspond serially with the information in the previously-generated file of member property data (containing member lengths, section properties, number of beam elements, etc.).

Data from these files are used to compute terms of the type $X^* M\ Y$ and $X^* K\ Y$ for use in the sweep-out procedure (Eqs.14 and 13), and to evaluate static equivalent loadings, $F = \omega^2\ M\ Y$, during the iterative solution process.

To reduce access time and computer costs, all data files are transmitted to and from secondary data storage units in large physical records (usually several thousand words per record, for large structures), each record containing data associated with many members. For problems of small size (e.g., structures containing only a few hundred members), it is often the case that the entire member property data file and the current modal approximation, Y, may be continuously retained in core storage as the succession of previously-computed modes, X_1, X_2, ... are transmitted into core for use in evaluating $X_1^* M\ Y$, $X_2^* M\ Y$, ...

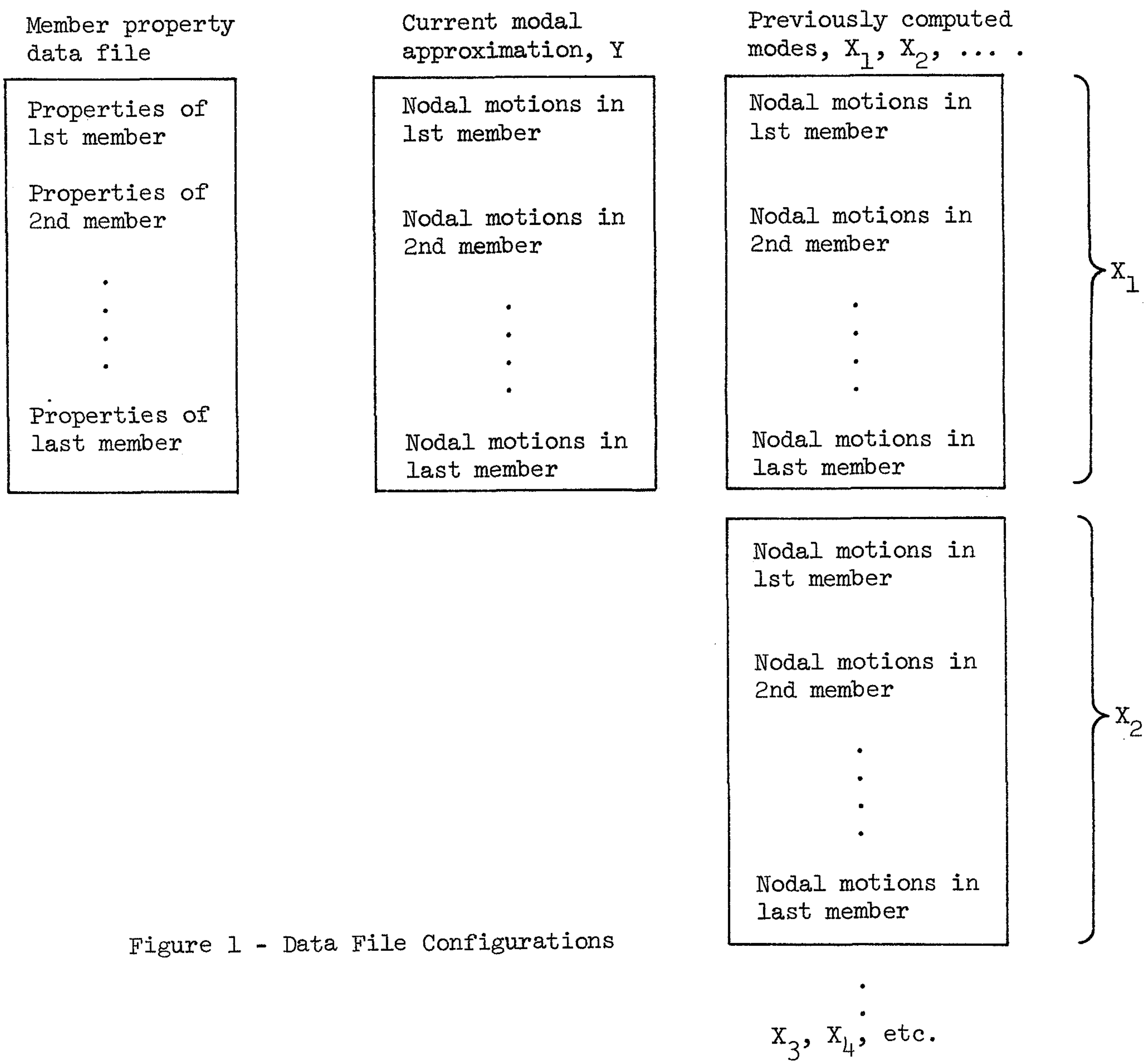


Figure 1 - Data File Configurations

3.3 EVALUATION OF X^*MY AND X^*KY

The mass and stiffness matrices, M and K, associated with large finite element networks are usually very sparse; accordingly, it is essential to the attainment of low computer execution costs that trivial arithmetic operations be avoided. Before discussing the computational procedure, some of the properties of M and K will be noted. Where T_k and V_k are the kinetic and potential energies of the k-th element, and the system contains r elements,

$$T = \sum_{k=1}^{r} T_k \text{ , where } T_k = \frac{1}{2} \dot{U}^* M_k \dot{U} \tag{17}$$

$$V = \sum_{k=1}^{r} V_k \text{ , where } V_k = \frac{1}{2} U^* K_k U \tag{18}$$

From the above equations, M and K are identified as follows:

$$M = \sum_{k=1}^{r} M_k \tag{19}$$

$$K = \sum_{k=1}^{r} K_k \quad . \tag{20}$$

In the frame dynamics program, polynomials of arbitrary order are used as displacement functions for each individual uniform straight beam element (the number of terms used in particular applications is specified in the input data); however, for simplicity of explanation, it will be assumed throughout the following discussion that the lateral bending displacements of the central bending axis in each principal plane are represented by cubic polynomials, and that axial extension and twisting are represented by linear functions. In this case, a unique relation exists between the coefficients of the 12 displacement functions and the 12 motion components of the two node points interconnected by the element (3 displacements and 3 rotations at each node), and it is convenient to use node point motion components explicitly as system generalized coordinates; that is, where u^i is a vector containing the 6 motion components of the i-th node point,

$$U = \begin{bmatrix} u^1 \\ u^2 \\ \cdot \\ \cdot \\ u^n \end{bmatrix} \tag{21}$$

It is assumed in the following discussion that the k-th element interconnects nodes i and j. It will be convenient to associate with each uniform beam element an "element reference frame", the 1 and 2 axes of which lie in principal bending planes, with the 3-axis directed along the elastic central axis from node i towards node j. The direction-p displacement components of nodes i and j (with respect to the k-th element reference frame) will be represented by the symbols $\bar{u}_p^i$ and $\bar{u}_p^j$, for p=1,2,3. For p=4,5, and 6, $\bar{u}_p^i$ and $\bar{u}_p^j$ will represent the direction p-3 rotation components of nodes i and j. For this choice of reference frame,

$$T_k = \frac{1}{2}\left[(\dot{\bar{u}}^i)^*(\dot{\bar{u}}^j)^*\right]\begin{bmatrix} \bar{M}_k^{ii} & \bar{M}_k^{ij} \\ (\bar{M}_k^{ij})^* & \bar{M}_k^{jj} \end{bmatrix}\begin{bmatrix} \dot{\bar{u}}^i \\ \dot{\bar{u}}^j \end{bmatrix} \tag{22}$$

In the above equations,

$$\bar{u}^i = \begin{bmatrix} \bar{u}_1^i \\ \bar{u}_2^i \\ \cdot \\ \cdot \\ \cdot \\ \bar{u}_6^i \end{bmatrix}, \text{ and } \bar{u}^j = \begin{bmatrix} \bar{u}_1^j \\ \bar{u}_2^j \\ \cdot \\ \cdot \\ \cdot \\ \bar{u}_6^j \end{bmatrix} \tag{23}$$

For simple uniform beams $\bar{M}^{ii}$, $\bar{M}^{ij}$, and $\bar{M}^{jj}$ have the following special forms, if the line of mass centers coincides with the elastic central axis.

$$\bar{M}_k^{ii} = \frac{\mu \ell}{420} \begin{bmatrix} 156 & 0 & 0 & 0 & 22\ell & 0 \\ 0 & 156 & 0 & -22\ell & 0 & 0 \\ 0 & 0 & 140 & 0 & 0 & 0 \\ 0 & -22\ell & 0 & 4\ell^2 & 0 & 0 \\ 22\ell & 0 & 0 & 0 & 4\ell^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 140\frac{\rho_3}{\mu} \end{bmatrix} \tag{24}$$

$$\bar{M}_k^{jj} = \frac{\mu \ell}{420} \begin{bmatrix} 156 & 0 & 0 & 0 & -22\ell & 0 \\ 0 & 156 & 0 & 22\ell & 0 & 0 \\ 0 & 0 & 140 & 0 & 0 & 0 \\ 0 & 22\ell & 0 & 4\ell^2 & 0 & 0 \\ -22\ell & 0 & 0 & 0 & 4\ell^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 140\frac{\rho_3}{\mu} \end{bmatrix} \tag{25}$$

$$\bar{M}_k^{ij} = \frac{\mu \ell}{420} \begin{bmatrix} 54 & 0 & 0 & 0 & -13\ell & 0 \\ 0 & 54 & 0 & 13\ell & 0 & 0 \\ 0 & 0 & 70 & 0 & 0 & 0 \\ 0 & -13\ell & 0 & -3\ell^2 & 0 & 0 \\ 13\ell & 0 & 0 & 0 & -3\ell^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 70\frac{\rho_3}{\mu} \end{bmatrix} \quad (26)$$

In the above equations, ℓ is the length of the element, μ is mass per unit length, and ρ_3 is mass moment of inertia about the central (3) axis. Other terms appear if the line of mass centers does not coincide with the elastic central axis, or if rotary inertia associated with lateral bending is taken into account.

If node i lies on the interior of a member, interconnecting a pair of the beam elements into which the member is subdivided (that is, node i is not a "joint" interconnecting two or more non-colinear members), we choose $u^i = \bar{u}^i$. However if node i and/or j are joints, the components of u^i and u^j may be defined with respect to reference frames other than the one associated with the k-th element, in which case

$$\bar{u}^i = Q_k^i \, u^i \text{ and } \bar{u}^j = Q_k^j \, u^j \; ,$$

where

$$Q_k^i = \begin{bmatrix} R_k^i & 0 \\ 0 & R_k^i \end{bmatrix} \quad \text{and} \quad Q_k^j = \begin{bmatrix} R_k^j & 0 \\ 0 & R_k^j \end{bmatrix}. \quad (27)$$

R_k^i and R_k^j are 3 x 3 matrices of direction cosines specifying the orientation of the k-th element reference frame relative to the reference frames with respect to which u^i and u^j, respectively, are defined (the program allows arbitrary orientation of individual joint reference systems, to permit specification of "oblique" restraint). Substitution of Eqs. 27 into 22 gives

$$T_k = \frac{1}{2} \left[(u^i)^*(u^j)^*\right] \begin{bmatrix} M_k^{ii} & M^{ij} \\ (M_k^{ij})^* & M^{jj} \end{bmatrix} \begin{bmatrix} u^i \\ u^j \end{bmatrix}, \tag{28}$$

where

$$\begin{bmatrix} M_k^{ii} & M_k^{ij} \\ (M_k^{ij})^* & M_k^{jj} \end{bmatrix} = \begin{bmatrix} (Q_k^i)^* & 0 \\ 0 & (Q_k^j)^* \end{bmatrix} \begin{bmatrix} \bar{M}_k^{ii} & \bar{M}_k^{ij} \\ (\bar{M}_k^{ij})^* & \bar{M}_k^{jj} \end{bmatrix} \begin{bmatrix} Q_k^i & 0 \\ 0 & Q_k^j \end{bmatrix}. \tag{29}$$

From the preceding equation it is evident that the form of M_k is as follows (continuing the assumption that the k-th element interconnects nodes i and j):

$$M_k = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ & M_k^{ii} & M^{ij} & \cdot \\ & & \cdot & \cdot \\ & & M_k^{jj} & \cdot \\ \text{symmetric} & & & \cdot \end{bmatrix} \tag{30}$$

M_k^{ii}, M_k^{ij}, M_k^{ji}, and M_k^{jj} are the only non-zero 6 x 6 submatrices of M_k. Accordingly, if X and Y are considered as arrays of 6-vectors,

$$X = \begin{bmatrix} x^1 \\ x^2 \\ \cdot \\ \cdot \\ x^n \end{bmatrix}, \qquad Y = \begin{bmatrix} y^1 \\ y^2 \\ \\ \\ y^n \end{bmatrix}, \tag{31}$$

$$X^* M\, Y = \sum_{k=1}^{r} X^* M_k\, Y = \sum_{k=1}^{r} m_k\ , \tag{32}$$

where

$$m^k = \left[(x^i)^*(x^j)^*\right] \begin{bmatrix} M_k^{ii} & M_k^{ij} \\ (M_k^{ij})^* & M_k^{jj} \end{bmatrix} \begin{bmatrix} y^i \\ y^j \end{bmatrix} \quad . \tag{33}$$

Where $\bar{x}^i = Q_k^i\, x^i$, $\bar{y}^i = Q_k^i\, y^i$, etc., Eq. 29 may be used to re-write Eq. 33 as follows.

$$m^k = \left[(\bar{x}^i)^*(\bar{x}^j)^*\right] \begin{bmatrix} \bar{M}_k^{ii} & \bar{M}_k^{ij} \\ (\bar{M}_k^{ij})^* & \bar{M}_k^{jj} \end{bmatrix} \begin{bmatrix} \bar{y}^i \\ \bar{y}^j \end{bmatrix} \tag{34}$$

The routine used in the program to compute terms of the type X^*MY reads information from three previously-generated files containing (1) member property data from which the terms of $\bar{M}_k^{ii}$, $\bar{M}_k^{ij}$ and $\bar{M}_k^{jj}$ are readily determined, (2) the nodal motions corresponding to Y, and (3) nodal motions associated with all previously-computed modes, X_1, X_2, ... For each member, in succession, the quantities m_k are computed for each beam element into which the member is subdivided. In evaluating m_k, as indicated by Eq. 34, account is taken of the sparsity of $\bar{M}_k^{ii}$, $\bar{M}_k^{ij}$, and $\bar{M}_k^{jj}$ to avoid trivial arithmetic operations. As indicated by Eq. 32, X^*MY is the summation of all the m_k's.

Terms of the type X^*KY may be evaluated similarly.

The program contains routines for calculating the c_i's in Eq. 14 as either X^*MY/X^*MX or X^*KY/X^*KX. The two procedures usually are equally accurate and require the same computer execution time.

3.4 EVALUATION OF SEQUENCES OF MODAL APPROXIMATIONS, Y^2, Y^3, etc.

The procedure used to compute solutions to Eq. 11 is outlined below. In connection with this discussion, it is important to note that a distinction, significant for computational purposes, is made between the frame's "joints", which generally interconnect two or more non-colinear members, and other nodes lying entirely within the interior of a particular member, interconnecting the colinear beam elements into which the member is subdivided.

1. Temporarily assuming all joints to be completely restrained, the loading $F = \omega^2 MY^j$ is applied. For each member, a straightforward transfer matrix procedure is used to compute the motions of all interior nodes and the forces and moments exerted by the member on its two terminal joints. In making these calculations for an individual member, the inertia forces and moments acting on all beam elements contained in the member are evaluated using information from the previously-generated file of member property data and the file containing the previous modal approximation, Y^j. This "fixed joint" solution is transmitted to a secondary data storage unit in the same format as the previously-discussed files containing Y^j and the X_i's. As these calculations are made for successive members, a summation is formed of the point forces and moments exerted on each joint by all members connected to the joint.

2. The joint restraints imposed in connection with the above calculations are released, and the joint motions produced by the point forces and moments acting on the joints due to the "fixed joint" loading are evaluated. These calculations are carried out as described in Reference 1 using the previously-generated data file serving essentially the same function as a system flexibility matrix. For purposes of computing these "free joint" solutions, each member is treated as a single

element, since no loads are applied at interior nodes. Accordingly these static analysis problems are usually of much lower order than the vibration eigenproblem, since the joint motions are the only unknowns.

3. For each member, all node point motion components corresponding to the "free-joint" solution computed in the preceeding step are evaluated and added to the "fixed joint" motions computed in Step 1 above to form Y^{j+1}. After computing the generalized stiffness as $F^* Y^{j+1}$ (where F was evaluated as $\omega^2 MY^j$), Y^{j+1} is normalized to obtain unit generalized mass.

3.5 CONVERGENCE

In most applications, it usually is easy to visualize static loadings comprised of a few point forces and/or moments applied at joints which will produce good initial approximations of at least the first few modes of the structure. In such cases, convergence of the iterative solution procedure has been found to be rapid, with two to six iterations usually sufficient to meet the convergence criteria indicated by Eq.16 (for $e = 10^{-5}$). From Eq.10 it is evident, however, that under some circumstances a relatively large number of iterations may be required. Suppose for example that the initial approximation, Y^o, computed for the k-th mode, X_k, is actually a good approximation of X_{k+1} (that is, all of the c_i^o's are very small except for c_{k+1}^o). In this case, successive Y^j's would first tend to a close approximation of X_{k+1} before finally converging to X_k. Since the computer execution time required for each iteration is usually quite small, this effect is usually of little practical significance.

All solutions necessarily contain some numerical error. A convenient physical interpretation of numerical error is obtained by considering the external forces required to cause the system to execute the motion, $U = \sin \omega t \; Y^{j+1}$.

$$M \ddot{U} + K U = \sin \omega t \; F = \sin \omega t (\omega^2 M \; Y^{j+1} - K \; Y^{j+1}) \; . \qquad (35)$$

Substitution of Eq. 5 into Eq. 35 gives

$$F = \omega^2 M(Y^{j+1} - Y^j). \qquad (36)$$

Section 4
EXAMPLES

Vibrational characteristics of the following examples were calculated with the previously discussed computer program. All figures were generated by the program using an SC-4020 plotter. Tables are presented for each example, summarizing computer execution cost data and frequency approximations computed at successive steps in the iteration process. The zeroth iteration refers to the initial approximation.

The summarized computer execution times consist preponderantly of magnetic tape operations required in manipulating various data files generated by the program. Accordingly, substantially lower computer costs may be expected when executing under efficient multi-program systems which bill input-output devices at much lower rates than the central processing unit. In executing the static solutions, two passes were made through an iterative accuracy improvement routine (analogous to the well-known near-inverse method) to assure a high degree of precision.

EXAMPLE 1

Figures 2, 3, 4 and 5 illustrate an undeformed perspective view and the first three modes of a 180 degree of freedom frame. Frequency approximations computed at successive steps in the iteration process are summarized in Table 1. Computer execution cost data is summarized in Table 2. The static loadings used to determine initial approximations to the first and second modes, respectively, were (1) a lateral force acting through joints at the top of the structure, and (2) a lateral force at the top, and an (oppositely directed) lateral force applied near the middle of the structure. To obtain an initial approximation of the third mode, a vertical force was applied through the central joints at the top of the structure. Planar motion was obtained by constraining an appropriate set of joint motion components.

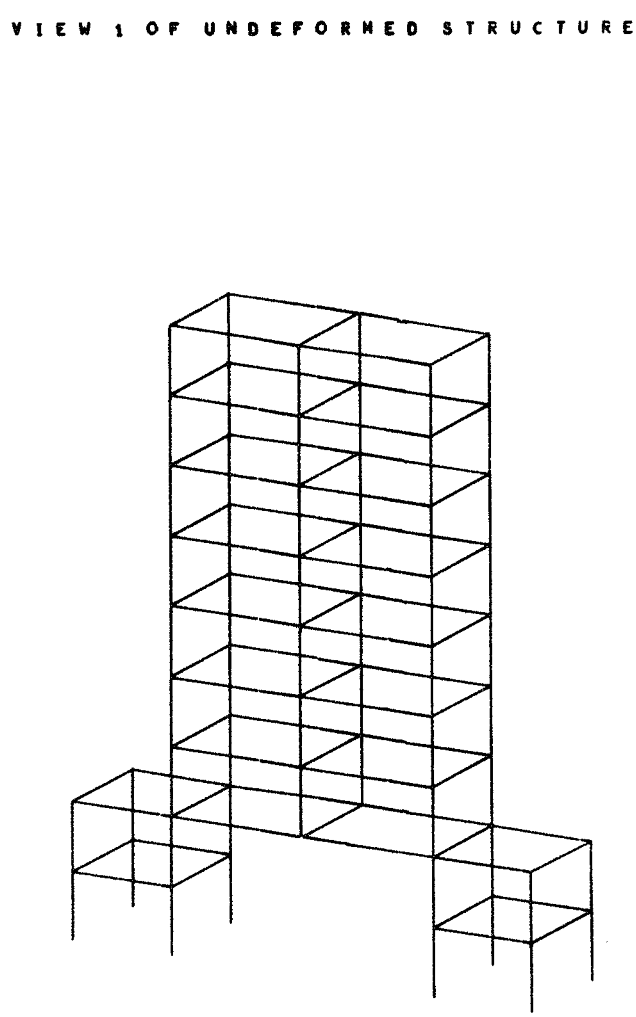


Fig. 2 - Undeformed View of 180 Degree of Freedom Frame

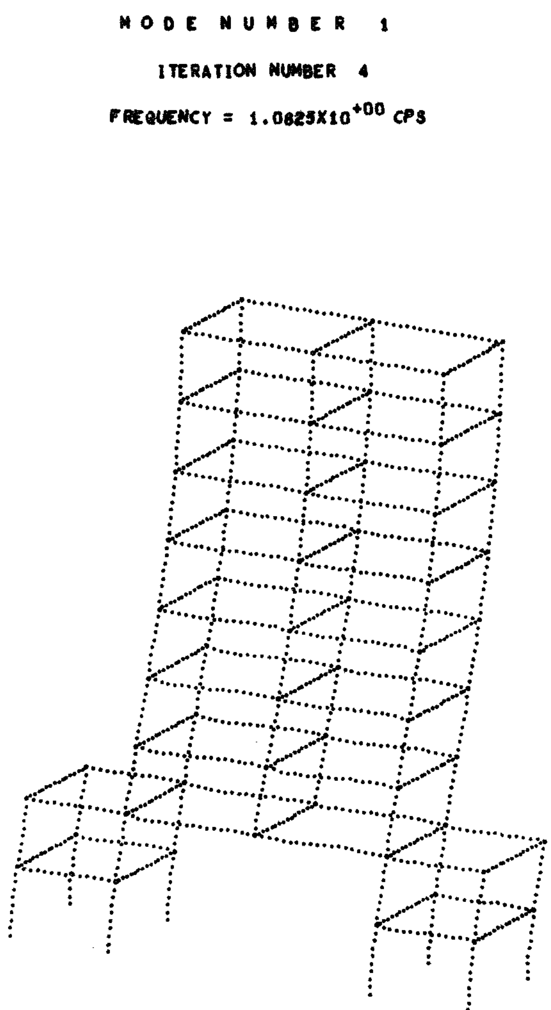


Fig. 3 - First Mode of 180 Degree of Freedom Frame

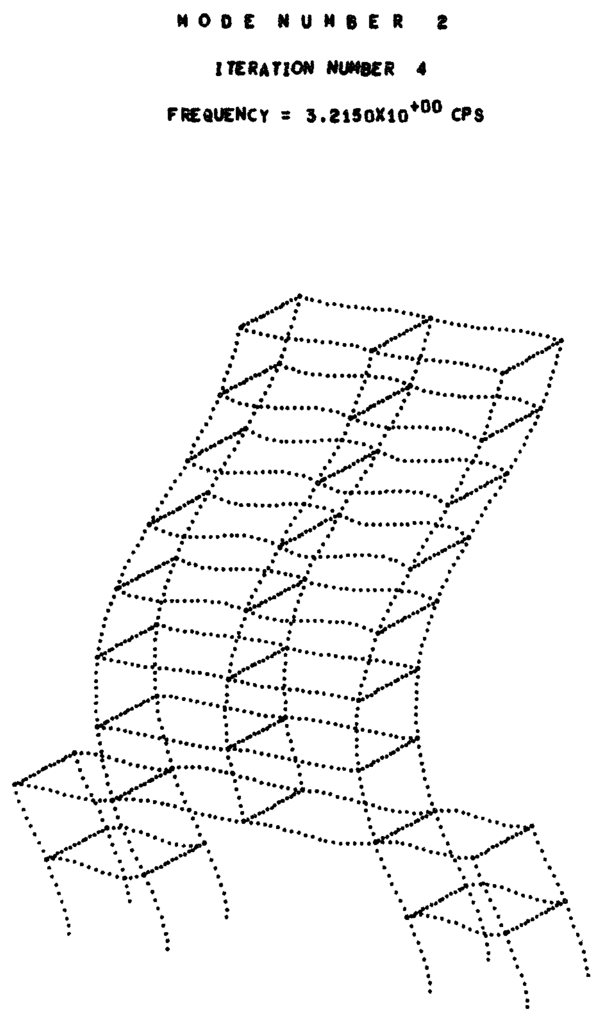


Fig. 4 - Second Mode of 180 Degree of Freedom Frame

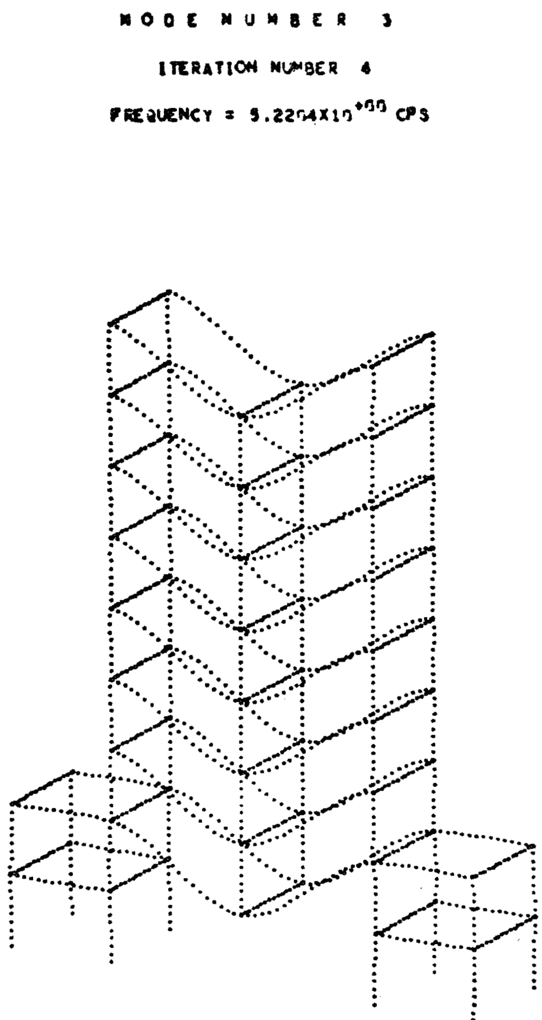


Fig. 5 - Third Mode of 180 Degree of Freedom Frame

Table 1

SUCCESSIVE FREQUENCY APPROXIMATIONS, EXAMPLE 1

Iteration	Mode 1	Mode 2	Mode 3
0	1.1582685	6.5587515	5.2523073
1	1.0832619	3.3580690	5.2204857
2	1.0826119	3.2197762	5.2204747
3	1.0826039	3.2152917	5.2204726
4	1.0826038	3.2150056	5.2204701
5	1.0826038	3.2149769	5.2204671

Table 2

IBM 7094 EXECUTION COSTS PER ITERATION, EXAMPLE 1

Mode	Total Execution Time per Iteration	Percent of Execution Time Consumed Executing Static Solutions
1	33 seconds ($1.37)	70
2	39 seconds ($1.62)	59
3	44 seconds ($1.83)	52

EXAMPLE 2

Figures 6 and 7 illustrate an undeformed perspective view and the first mode, respectively, of a 96-joint rectangular frame. Figure 8 illustrates an undeformed view orthogonal to that of Fig. 6. The second mode is shown on Fig. 9. Table 3 summarizes frequency approximations computed at successive steps in the iteration process. Table 4 summarizes computer execution cost data for the example.* The static loadings used to determine initial approximations to both modes were lateral forces acting in appropriate directions through joints at the top of the structure.

*Note that this solution was executed on the Univac 1108 computer under the EXEC II system.

VIEW 1 OF UNDEFORMED STRUCTURE

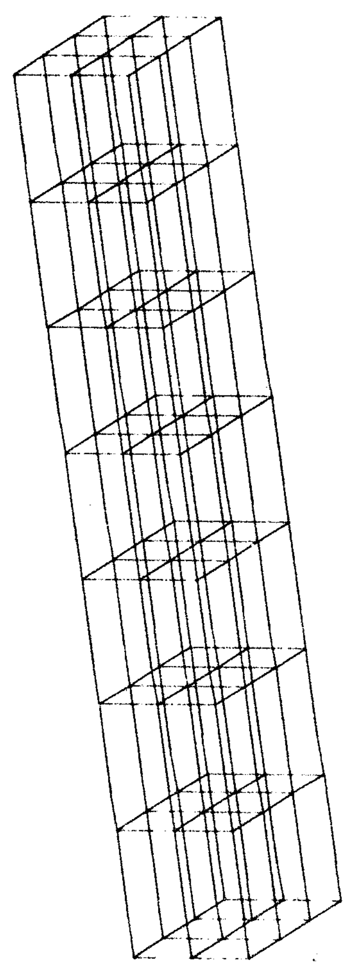
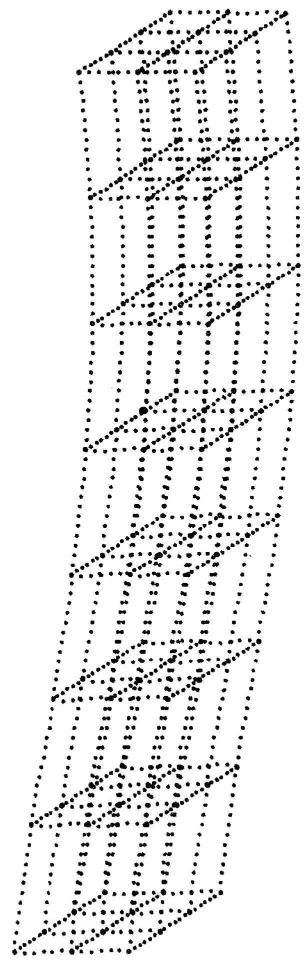


Fig. 6 - Undeformed View of 96-Joint Frame

MODE NUMBER 1 ITERATION 2



FREQUENCY = $.461365 \times 10^{+00}$ CPS

Fig. 7 - First Mode of 96-Joint Frame

VIEW 2 OF UNDEFORMED STRUCTURE

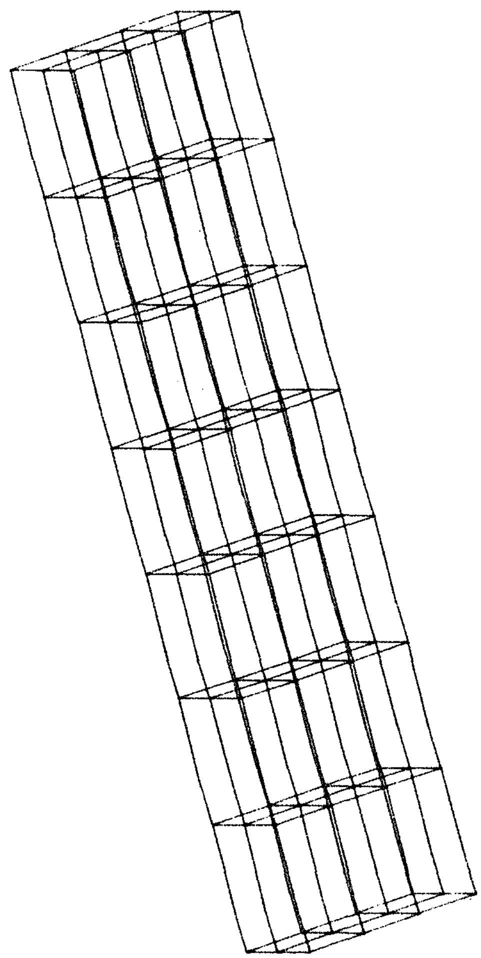


Fig. 8 - Undeformed View of 96-Joint Frame

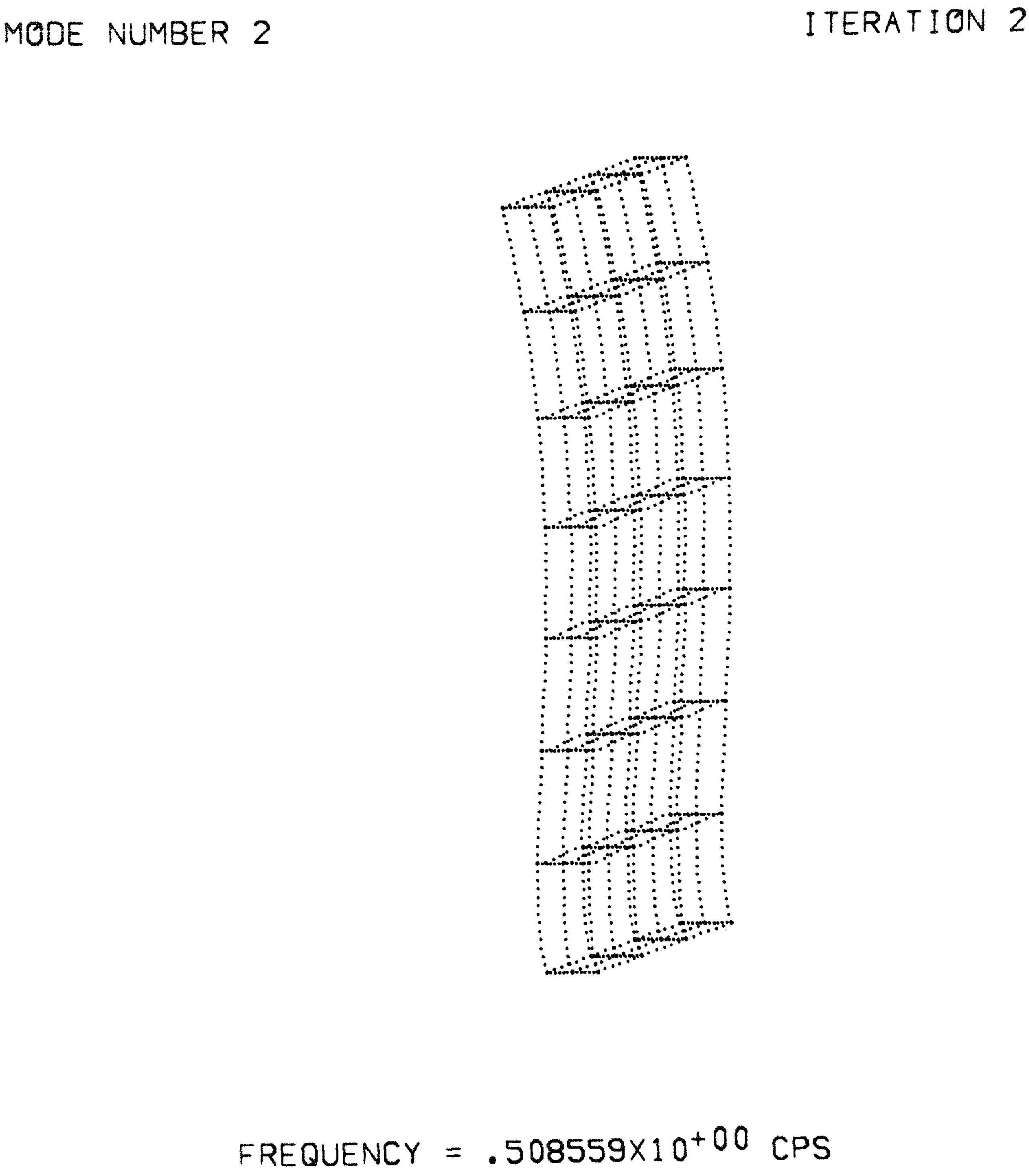


Fig. 9 - Second Mode of 96-Joint Frame

Table 3

SUCCESSIVE FREQUENCY APPROXIMATIONS, EXAMPLE 2

Iteration	Mode 1	Mode 2
0	0.50333833	0.54968914
1	0.46231770	0.50881150
2	0.46211027	0.50856023

Table 4

UNIVAC 1108, EXEC II SYSTEM, COSTS PER ITERATION, EXAMPLE 2

Mode	Total Execution Time per Iteration	Percent of Execution Time Consumed Executing Static Solutions
1	40 seconds ($6.40)	50
2	67 seconds ($10.72)	33

EXAMPLE 3

Figure 10 is a perspective view of the undeformed Saturn V umbilical tower. The structure is composed of 372 joints and 944 members. Figures 11, 12 and 13 illustrate the frame's undeformed configuration and the first two modes in the vehicle-tower plane (direction-1). Figures 14, 15 and 16 show the undeformed configuration and the first two modes in the plane normal to the vehicle-tower plane (direction-2). Frequency approximations computed at successive steps in the direction-2 solution process are summarized in Table 5. Corresponding computer execution cost data is given in Table 6.

For each direction, the static loadings used to determine initial approximations to the first and second modes, respectively, were (1) lateral forces acting through joints at the top of the structure, and (2) a lateral force at the top, and an (oppositely directed) lateral force applied near the middle of the structure. Planar motion was obtained by constraining an appropriate set of joint motion components.

The effect of non-structural mass was included in the formulation by appropriately increasing the mass density of the peripheral members at each floor level of the structure.

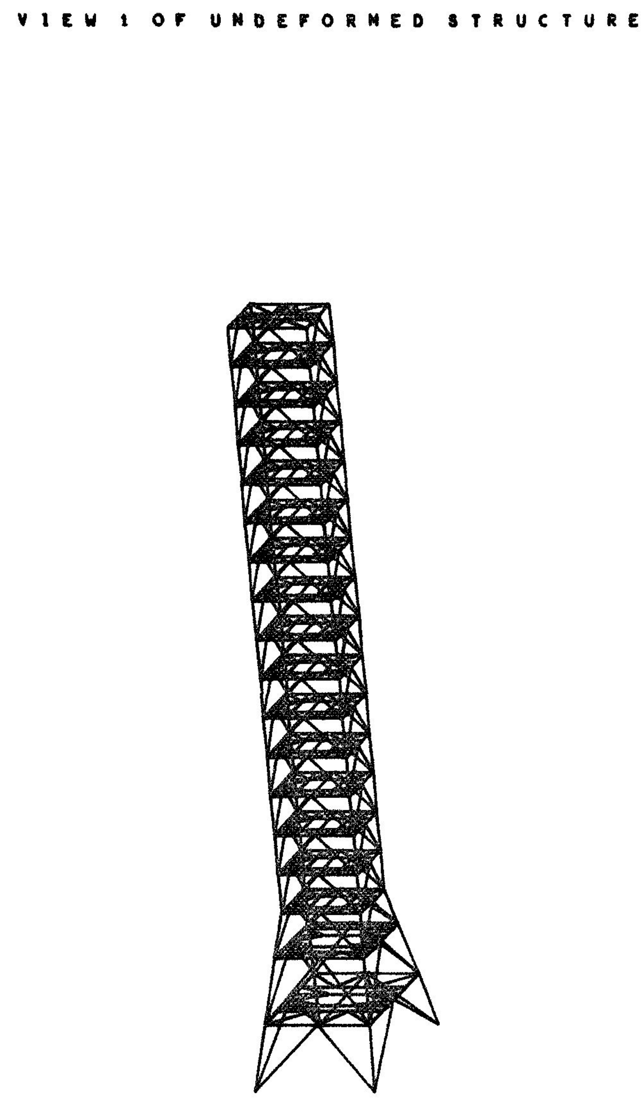


Fig. 10 - Perspective View of Launcher Umbilical Tower

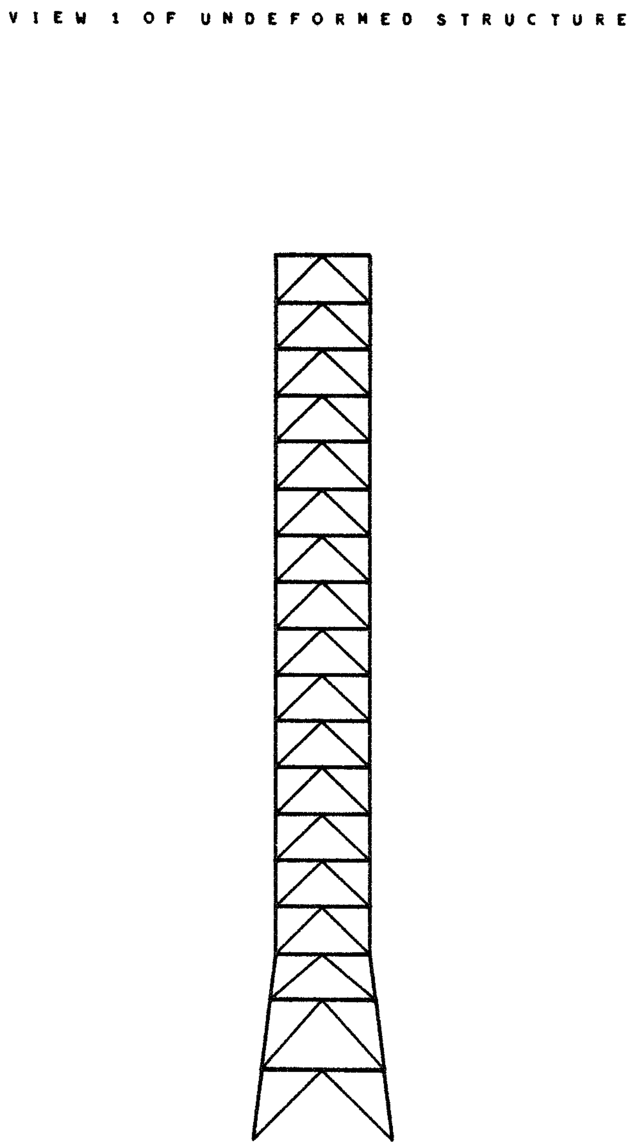


Fig. 11 - Undeformed View of Launcher Umbilical Tower (Direction-1)

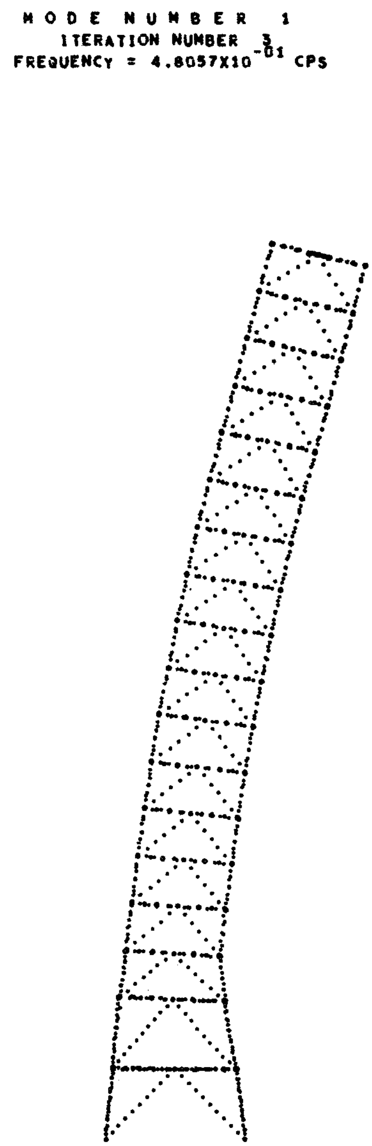


Fig. 12 - First Mode of Launcher Umbilical Tower (Direction-1)

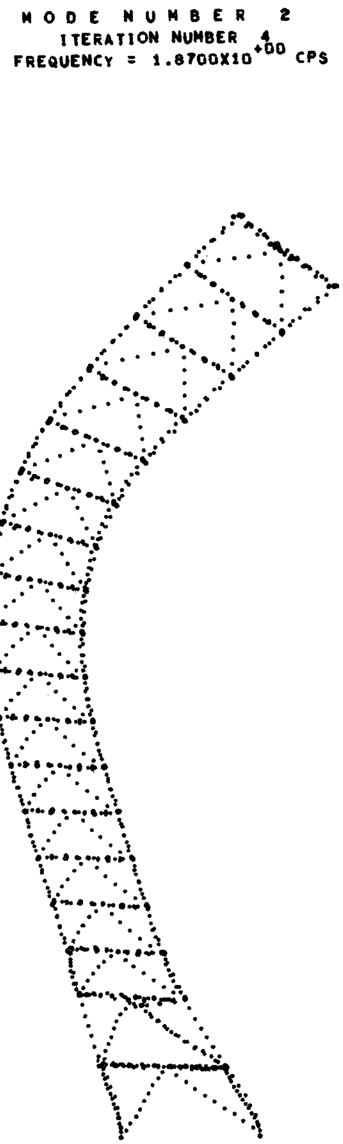


Fig. 13 - Second Mode of Launcher Umbilical Tower (Direction-1)

VIEW 1 OF UNDEFORMED STRUCTURE

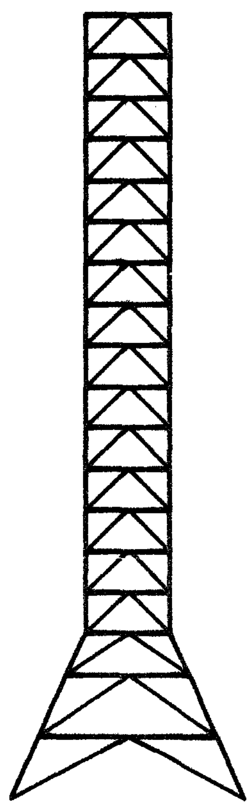


Fig. 14 - Undeformed View of Launcher Umbilical Tower (Direction-2)

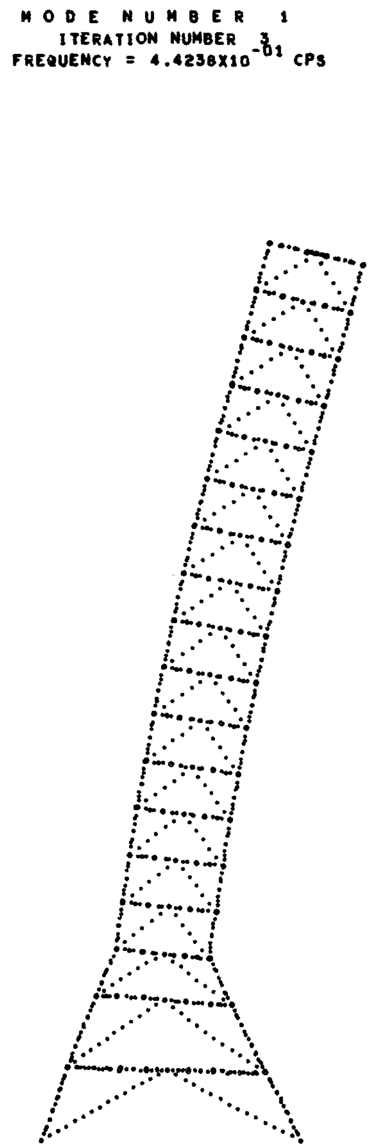


Fig. 15 - First Mode of Launcher Umbilical Tower (Direction-2)

Fig. 16 - Second Mode of Launcher Umbilical Tower (Direction-2)

Table 5

SUCCESSIVE FREQUENCY APPROXIMATIONS,
LAUNCHER UMBILICAL TOWER, DIRECTION-2

Iteration	Mode 1	Mode 2
0	0.45528928	2.7176277
1	0.44239758	2.1161577
2	0.44238652	2.1143194
3	0.44238651	2.1142560
4	–	2.1142523

Table 6

IBM 7094 COST PER ITERATION,
LAUNCHER UMBILICAL TOWER ANALYSIS

Mode	Total Execution Time per Iteration*	Percent of Execution Time Consumed Executing Static Solutions
1	173 seconds ($26.70)	76
2	289 seconds ($44.60)	46

*These times include two passes through an iterative accuracy improvement routine to assure a high degree of precision. Magnetic tape access and transmissions account for all but a very small fraction of the indicated IBM 7094 execution time; accordingly, much lower costs would be obtained on a system with high-speed random access devices (e.g., a Univac 1108 with FH 432 or 1782 drums).

Section 5
REFERENCES

1. Whetstone, W.D., "Computer Analysis of Large Linear Space Frames," LMSC/HREC A791019, Lockheed Missiles & Space Company, Huntsville Research & Engineering Center, Huntsville, Ala., 1968.

2. Hurty, W.C. and M.F. Rubenstein, Dynamics of Structures, Prentice-Hall, Englewood Cliffs, N.J., 1964, p.123.

APPENDIX

INPUT INSTRUCTIONS. MARCH 1969 VERSION OF FRAME DYNAMICS PROGRAM, FAMDAM, FOR VIBRATION ANALYSIS OF LARGE LINEAR FRAMES.

TABLE OF CONTENTS

GENERAL INSTRUCTIONS SECTION 1

THE INPUT DATA SEQUENCE FOR A SINGLE STRUCTURE IS OUTLINED BELOW. BY STACKING DATA DECKS SEQUENTIALLY, ANY NUMBER OF STRUCTURES CAN BE ANALYZED IN A SINGLE EXECUTION.

A. INTERMEDIATE DATA OUTPUT AND GENERAL SOLUTION INFORMATION CONTROL CARD

LIST= K1,K2,K3,K4,KIND,JTRGD

FORMAT(6I1)

IF NON-ZERO, K1,K2,K3, AND K4 CAUSE DATA FROM THE H, A, R, AND M-FILES, RESPECTIVELY, TO BE PRINTED. THIS DATA SHOULD NOT NORMALLY BE REQUESTED.

KIND=0 INDICATES THAT THIS IS THE INITIAL EXECUTION FOR THIS PROBLEM. KIND=1 INDICATES THAT THIS IS A CONTINUED SOLUTION. FOR A CONTINUED SOLUTION READ IN TITLE CARDS DISCUSSED BELOW THEN REFER DIRECTLY TO SECTION 10.

JTRGD.NE.0 INDICATES THAT THIS STRUCTURE IS EXTERNALLY UNSTABLE, IN WHICH CASE ARTIFICIAL CONSTRAINTS ARE APPLIED TO JOINT JTRGD ACCORDING TO THE INSTRUCTIONS IN SECTION 8. FOR THIS CASE SIX RIGID BODY MODES ARE COMPUTED FOR THE STRUCTURE. IF JTRGD=0 THE STRUCTURE IS ASSUMED EXTERNALLY STABLE.

B. TITLE CARDS. (AT LEAST ONE CARD MUST APPEAR)

ANY NUMBER OF TITLE CARDS CAN BE USED. INFORMATION APPEARING IN COLUMNS 2 THROUGH 73 OF THESE CARDS WILL APPEAR AT THE BEGINNING OF THE PRINTED OUTPUT. THE LAST TITLE CARD MUST HAVE A BLANK IN COLUMN 1, ALL PRECEDING CARDS (IF ANY) MUST HAVE A NON-ZERO INTEGER IN COLUMN 1.

C. LIBRARY DATA CONTROL CARD.

LIST= JT, NCOS, NSECT, NMAT, N6X6, INX

FORMAT(5I5, 5X, I5)

JT IS THE NUMBER OF JOINTS IN THE STRUCTURE. IN A MANNER INDICATED SUBSEQUENTLY, THE OTHER VARIABLES CONTROL INPUT OF DATA IN THE FOLLOWING CATEGORIES.

NCOS- MEMBER REFERENCE FRAME ORIENTATION SPECIFICATIONS.
NSECT- BEAM SECTION PROPERTIES
NMAT- MATERIAL CONSTANTS.
N6X6- DIRECTLY SPECIFIED GENERAL 6X6 BEAM STIFFNESS MATRICES.
INX- JOINT POSITION COORDINATES.

D. LIBRARY OF MATERIAL CONSTANTS.
SEE SECTION 2 FOR DETAILS.

E. JOINT POSITION COORDINATES AND JOINT REFERENCE FRAME IDENTIFICATION.
SEE SECTION 3 FOR DETAILS.

F. LIBRARY OF MEMBER REFERENCE FRAME ORIENTATION SPECIFICATIONS.
SEE SECTION 4 FOR DETAILS.

G. LIBRARY OF BEAM SECTION PROPERTIES.
SEE SECTION 5 FOR DETAILS.

H. LIBRARY OF DIRECTLY SPECIFIED GENERAL 6X6 BEAM STIFFNESS MATRICES.
SEE SECTION 6 FOR DETAILS.

I. LIBRARY OF JOINT REFERENCE FRAME ORIENTATION SPECIFICATIONS.
SEE SECTION 7 FOR DETAILS.

J. STRUCTURE DEFINITION DECK.
SEE SECTION 8 FOR DETAILS.

K. VIBRATION ANALYSIS CONTROL CARDS.
SEE SECTION 9 FOR DETAILS

L. CONTINUED SOLUTION DATA (ONLY WHEN KIND.NE.0)
SEE SECTION 10 FOR DETAILS

M. SOLUTION CONTROL PARAMETERS
SEE SECTION 11 FOR DETAILS

LIBRARY OF MATERIAL CONSTANTS SECTION 2

THERE ARE NMAT CARDS (AT LEAST ONE) IN THIS DATA GROUP. EACH CARD SPECIFIES THE CONSTANTS ASSOCIATED WITH A PARTICULAR MATERIAL. INPUT OF THESE CARDS IS CONTROLLED BY THE FOLLOWING LIST AND FORMAT.

```
LIST= (E(I), G(I), SPWT(I), I=1,NMAT)

FORMAT(3E12.3)
```

E(I), G(I), AND SPWT(I) ARE THE MODULUS OF ELASTICITY, SHEAR MODULUS, AND SPECIFIC WEIGHT, RESPECTIVELY, OF THE I-TH MATERIAL.

NMAT MUST NOT EXCEED 4.

JOINT POSITION COORDINATES IN THE GLOBAL REFERENCE FRAME SECTION 3

DEPENDING ON THE VALUE OF THE PREVIOUSLY READ CONTROL VARIABLE INX, EITHER OF TWO PROCEDURES CAN BE USED.

IF INX=7, JOINT POSITION COORDINATE INPUT IS CONTROLLED BY THE FOLLOWING LIST AND FORMAT

```
LIST= ((X(I,J),I=1,3),J=1,JT)

FORMAT(6F10.3),
```

WHERE X(I,J) IS THE DIRECTION-I POSITION COORDINATE (GLOBAL RECTANGULAR REFERENCE FRAME) OF THE J-TH JOINT.

IF INX IS NOT EQUAL TO 7, THIS DATA DECK IS COMPOSED OF JT CARDS (ONE FOR EACH JOINT) WITH INPUT OF EACH CARD CONTROLLED BY THE FOLLOWING LIST AND FORMAT

```
LIST= J, (X(I,J),I=1,3)

FORMAT( I5, 5X, 3E15.8 ),
```

J IS THE IDENTIFYING INDEX NUMBER ASSIGNED TO THE JOINT, AND THE X(I,J)'S ARE THE JOINT'S POSITION COORDINATES IN THE GLOBAL RECTANGULAR REFERFNCE FRAME.

LIBRARY OF MEMBER REFERENCE FRAME ORIENTATION SPECIFICATIONS — SECTION 4

THERE ARE NCOS CARDS (AT LEAST ONE) IN THIS DATA GROUP. EACH CARD CONTAINS ONE COMPLETE SPECIFICATION WHICH IS, IN MOST CASES, APPLICABLE TO MANY MEMBERS. INPUT OF EACH CARD IS CONTROLLED BY THE FOLLOWING LIST AND FORMAT.

```
LIST=   I, NM, NG, R(NM,NG)

FORMAT(I3, I3,I1, E13.3).
```

I IS THE IDENTIFYING INDEX NUMBER OF THIS SPECIFICATION.

THE 3-AXIS OF EACH MEMBER REFERENCE FRAME IS DIRECTED FROM THE MEMBER 'ORIGIN' THROUGH THE MEMBER 'TERMINUS' (THE ORIGIN IS THE MEMBER END POINT CONNECTED TO THE LOWER-NUMBERED JOINT).

R(NM,NG) IS THE COSINE OF THE ANGLE BETWEEN THE NM-TH AXIS OF THE MEMBER FRAME AND THE NG-TH AXIS OF THE GLOBAL RECTANGULAR FRAME. SINCE THE ORIENTATION OF THE 3-AXIS OF EACH MEMBER REFERENCE FRAME IS COMPLETELY DETERMINED BY THE MEMBER'S END POINT LOCATIONS, NM MUST BE EITHER 1 OR 2, ALTHOUGH NG MAY BE EITHER 1,2, OR 3.

IF THE COSINE OF THE ANGLE BETWEEN MEMBER AXIS (3-NM) AND GLOBAL AXIS NG IS NEGATIVE, A MINUS SIGN MUST BE PLACED IN COLUMN 5, IMMEDIATELY PRECEDING NM. FOR EXAMPLE, IF WE SPECIFY R(2,3) = .7 AND R(1,3) IS NEGATIVE, A MINUS SIGN IS PLACED IN COLUMN 5.

CARE MUST BE TAKEN TO ENSURE UNIQUE SPECIFICATIONS. FOR EXAMPLE, IF A MEMBER'S END POINT LOCATIONS ARE SUCH THAT THE MEMBER 3-AXIS IS PARALLEL TO THE GLOBAL 2-AXIS, THE MEMBER FRAME ORIENTATION CANNOT BE DETERMINED BY SPECIFICATIONS THAT R(1,2) OR R(2,2) ARE ZERO.

LIBRARY OF BEAM SECTION PROPERTIES SECTION 5

IF NSECT=0, THIS DATA GROUP IS NOT PRESENT (DO NOT INSERT A BLANK CARD), OTHERWISE, THERE ARE NSECT ENTRIES IN THE LIBRARY. USUALLY, EACH ENTRY APPLIES TO MANY DIFFERENT MEMBERS. INPUT OF THE FIRST CARD OF EACH ENTRY IS CONTROLLED BY THE FOLLOWING LIST AND FORMAT.

```
LIST=  N, TYPE, MAT, (D(I),I=1,7)

FORMAT(I2, 1X, A4, I3, 7E10.3)
```

N (AN INTEGER BETWEEN 1 AND NSECT) IS THE INDEX IDENTIFYING THE ENTRY. 'TYPE' IS A 4 CHARACTER ALPHAMERIC WORD IDENTIFYING THE SECTION TYPE. 'TYPE' MUST BE LEFT-ADJUSTED IN THE A4 FIELD (THAT IS, THE FIRST LETTER OF THE WORD MUST APPEAR IN COLUMN 4). MAT IDENTIFIES THE APPLICABLE SET OF MATERIAL CONSTANT DATA. THE D ARRAY SPECIFIES SECTION PROPERTIES, AS SUMMARIZED IN THE FOLLOWING TABLE.

TYPE	D(1)	D(2)	D(3)	D(4)	D(5)	D(6)	D(7)
GIVN	I1	ALPHA1	I2	ALPHA2	AREA	C	C1
BOX	B1	T1	B2	T2			
WFL	B1	T1	B2	T2	B3	T3	
TEE	B1	T1	B2	T2			
ANG	B1	T1	B2	T2			
CHN	B1	T1	B2	T2	B3	T3	
ZEE	B1	T1	B2	T2	B3	T3	
TUBE	R(INNER)	R(OUTER)					

I1 AND ALPHA1 ARE ASSOCIATED WITH BENDING IN PLANE 2-3.
I2 AND ALPHA2 ARE ASSOCIATED WITH BENDING IN PLANE 1-3.
I1= MOMENT OF INERTIA ABOUT MEMBER AXIS-1 (A PRINCIPAL AXIS).
I2= MOMENT OF INERTIA ABOUT MEMBER AXIS-2 (THE OTHER PRINCIPAL AXIS).
ALPHA1 AND ALPHA2 ARE TRANSVERSE SHEAR DEFLECTION CONSTANTS ASSOCIATED WITH I1 AND I2, RESPECTIVELY (SEE TIMOSHENKO 'STRENGTH OF MATERIALS', PART ONE, PAGE 170). AREA= CROSS-SECTIONAL AREA. C= UNIFORM TORSION CONSTANT (TORQUE/TWIST PER UNIT LENGTH), AND C1= NON-UNIFORM TORSION CONSTANT (SEE TIMOSHENKO 'STRENGTH OF MATERIALS', PART 2, PAGES 255-273).

THE FOLLOWING CARD IS USED ONLY FOR 'GIVN' SECTIONS. IT IS NOT USED FOR ANY OTHER SECTION TYPE.

LIST= Z1, Z2, THETA

FORMAT(3E10.3)

EACH MEMBER 'ORIGIN' COINCIDES WITH THE SECTION CENTROID, AND THE 3-AXIS OF EACH LOCAL MEMBER REFERENCE FRAME IS DIRECTED FROM THE MEMBER 'ORIGIN' ALONG THE CENTRAL AXIS. THE 1 AND 2 AXES OF THE MEMBER FRAME ARE NOT REQUIRED TO COINCIDE WITH SECTION PRINCIPAL AXES. Z1 AND Z2 ARE THE POSITION COORDINATES IN THE MEMBER REFERENCE FRAME OF THE SHEAR CENTER. THETA IS THE ANGLE, IN RADIANS, MEASURED POSITIVE CLOCKWISE ABOUT THE MEMBER CENTRAL (3) AXIS FROM THE 1-AXIS OF THE MEMBER FRAME TO PRINCIPAL AXIS-1. NORMALLY THETA IS SET EQUAL TO ZERO.

LIST= MASS,RHO1,RHO2,RHO3

FORMAT(6E10.3)

MASS= MASS PER UNIT LENGTH OF THE MEMBER
RHO1= MASS MOMENT OF INERTIA OF THE MEMBER ABOUT ITS 1 AXIS
RHO2= MASS MOMENT OF INERTIA OF THE MEMBER ABOUT ITS 2 AXIS
RHO3= MASS MOMENT OF INERTIA OF THE MEMBER ABOUT ITS 3 AXIS

LIBRARY OF DIRECTLY-SPECIFIED MEMBER STIFFNESS MATRICES — SECTION 6

IF N6X6=0, THIS DATA GROUP IS NOT PRESENT (DO NOT INCLUDE A BLANK CARD). OTHERWISE, N6X6 MATRICES ARE READ, CONTROLLED BY THE FOLLOWING LIST AND FORMAT.

```
LIST= (((S(I,J,K),J=1,6),I=1,6),K=1,N6X6)

FORMAT(6E10.3)
```

S(I,J,K) IS THE ELEMENT IN THE I-TH ROW AND J-TH COLUMN OF THE K-TH MATRIX. THE PROGRAM USES ONLY THE LOWER TRIANGULAR PART OF EACH MATRIX, TO ENSURE SYMMETRY.

LIBRARY OF JOINT REFERENCE FRAME ORIENTATION SPECIFICATIONS — SECTION 7

IF JRF=0, THIS DATA GROUP IS NOT PRESENT (DO NOT INCLUDE A BLANK CARD). OTHERWISE THERE ARE JRF ENTRIES (ONE PER CARD) IN THE LIBRARY. INPUT OF EACH ENTRY IS CONTROLLED BY THE FOLLOWING LIST AND FORMAT

```
LIST= K, J1,Q(3,J1),J2,Q(3,J2),J3SIGN,  NL,NG,NSIGN,Q(NL,NG)

FORMAT(I2,I3,E15.5,I5,E15.5,I5,6X,2I1,I2,E15.5)
```

K (AN INTEGER BETWEEN 1 AND JRF) IS THE INDEX IDENTIFYING THE ENTRY. THE Q MATRIX (3X3) REPRESENTS THE ORIENTATION OF A JOINT REFERENCE FRAME RELATIVE TO THE GLOBAL FRAME. FROM THE INFORMATION GIVEN ON THE ABOVE DATA CARD, THE PROGRAM COMPUTES A Q MATRIX WHICH MAY APPLY TO SEVERAL JOINTS. Q(I,J) IS THE COSINE OF THE ANGLE BETWEEN THE I-TH AXIS OF THE JOINT FRAME AND THE J-TH AXIS OF THE GLOBAL RECTANGULAR FRAME.

J1, Q(3,J1), J2, Q(3,J2), AND J3SIGN COMPLETELY THE ORIENTATION OF THE 3-AXIS OF THE JOINT FRAME. Q(3,J1) AND Q(3,J2) ARE ANY TWO DISTINCT ELEMENTS IN THE THIRD ROW OF Q, AND J3SIGN (+1 OR -1) GIVES THE SIGN OF THE THIRD ELEMENT.

Q(NL,NG) MAY GENERALLY BE ANY ELEMENT IN THE FIRST TWO ROWS OF Q. NSIGN (+1 OR -1) GIVES THE SIGN OF Q(3-NL,NG).

IN CHOOSING NL AND NG, CARE MUST BE TAKEN TO ENSURE A UNIQUE SPECIFICATION. FOR EXAMPLE, IF THE 3-AXIS OF THE JOINT FRAME WERE CHOSEN TO BE PARALLEL TO THE GLOBAL 2-AXIS (EG J1=1,J2=3, Q(3,J1)=.0, Q(3,J2)=.0,J3SIGN= +1), THEN THE ORIENTATION OF THE JOINT'S 1 AND 2 AXES IS NOT UNIQUELY DETERMINED BY SPECIFICATIONS THAT R(1,2) OR R(2,2) ARE ZERO.

STRUCTURE DEFINITION DECK — SECTION 8

THE STRUCTURE DEFINITION DECK IS COMPOSED OF JT SETS OF CARDS, ONE SET FOR EACH JOINT, ORDERED BY ASCENDING JOINT NUMBERS. INPUT OF THE FIRST CARD OF EACH SET IS CONTROLLED BY THE FOLLOWING LIST AND FORMAT.

```
LIST= (KC(K),K=1,6), NMEMS

FORMAT(6I1,I4)
```

IF KC(K) IS A NON-ZERO INTEGER, THE K-TH MOTION COMPONENT (WITH RESPECT TO THE LOCAL JOINT REFERENCE FRAME) OF THIS JOINT IS SPECIFIED AS A RESTRAINT CONDITION (K=1,2,3- DISPLACEMENTS, K=4,5,6- ROTATIONS). THE PROGRAM SETS ALL RESTRAINED JOINT MOTION COMPONENTS IDENTICALLY EQUAL TO ZERO.
NMEMS IS THE NUMBER OF MEMBERS CONNECTING THE JOINT TO HIGHER-NUMBERED JOINTS. IF NMEMS IS ZERO (AS IS OFTEN THE CASE) THERE ARE NO ADDITIONAL CARDS IN THE DATA SET ASSOCIATED WITH THIS JOINT. OTHERWISE, DATA DEFINING EACH OF THE NMEMS MEMBERS CONNECTING THIS JOINT TO HIGHER-NUMBERED JOINTS FOLLOWS. INPUT OF THE FIRST CARD ASSOCIATED WITH THE DEFINITION OF EACH MEMBER IS CONTROLLED BY THE FOLLOWING LIST AND FORMAT.

```
LIST= N1,N2,MKOS,MTYPE,MGROUP,NOFF,NSTA,IPLOT

FORMAT(8I5)
```

N1 AND N2 ARE THE CONNECTED JOINTS (N1 IS THE CURRENT JOINT NUMBER, AND N2 IS GREATER THAN N1). THE MEMBER'S 'ORIGIN' CONNECTS TO JOINT N1, ITS 'TERMINUS' TO JOINT N2. MKOS IS AN INTEGER BETWEEN 1 AND NCOS (SEE SECTION 1, CARD C, AND SECTION 4) SPECIFYING THE APPLICABLE ENTRY IN THE PREVIOUSLY-INPUT LIBRARY OF MEMBER REFERENCE FRAME ORIENTATION SPECIFICATIONS. THE 3 AXIS OF THE MEMBER REFERENCE FRAME IS DIRECTED FROM THE MEMBER ORIGIN THROUGH THE TERMINUS.

IF NOFF IS ZERO (OR BLANK), THE MEMBER'S END POINTS (CENTROIDS, FOR STRAIGHT BEAMS HAVING SECTION PROPERTIES DEFINED IN THE PREVIOUSLY-INPUT LIBRARY DISCUSSED IN SECTION 5) COINCIDE WITH THE JOINTS TO WHICH THEY ARE CONNECTED. IF NOFF IS A POSITIVE INTEGER, ONE OR BOTH OF THE MEMBER'S END POINTS ARE OFFSET FROM THE CONNECTED JOINTS BY RIGID LINKS, WITH BOTH THE JOINTS AND MEMBER END POINTS RIGIDLY EMBEDDED IN THE LINKS. IF (AND ONLY IF) NOFF IS A POSITIVE INTEGER, THE FOLLOWING CARD MUST APPEAR NEXT IN THE INPUT DATA DECK.

```
LIST= (DO1(I),I=1,3), (DT2(I),I=1,3)

FORMAT(6E10.3)
```

DO1(I) IS THE DIRECTION I POSITION COORDINATE (GLOBAL RECTANGULAR FRAME) OF JOINT N1 MINUS THE DIRECTION I COORDINATE OF THE MEMBER ORIGIN. DT2 GIVES THE POSITION COORDINATES OF JOINT N2 MINUS THE CORRESPONDING COORDINATES OF THE TERMINUS.

IF MTYPE=1,THIS MEMBER IS A STRAIGHT BEAM HAVING CROSS-SECTION PROPERTIES DEFINED BY THE MGROUP-TH ENTRY IN THE PREVIOUSLY-INPUT LIBRARY OF BEAM SECTION PROPERTIES. MGROUP IS IN THIS CASE AN INTEGER BETWEEN 1 AND NSECT (SEE SECTION 1, CARD 3, AND SECTION 5)

IF MTYPE=2, THE 6X6 STIFFNESS MATRIX OF THIS MEMBER IS READ IN DIRECTLY, ACCORDING TO THE FOLLOWING LIST AND FORMAT, IMMEDIATELY FOLLOWING THE PRECEEDING CARD(S) ASSOCIATED WITH THIS MEMBER.

```
LIST= ((S(I,J),J=1,6),I=1,6)

FORMAT(6E10.3)
```

S IS A MATRIX SUCH THAT P=SV, WHERE V IS A 6-VECTOR REPRESENTING THE MOTION OF THE MEMBER ORIGIN RELATIVE TO THE TERMINUS, AND P IS THE VECTOR OF FORCE AND MOMENT COMPONENTS ACTING ON THE ORIGIN (FOR I=1,2,3, THE P(I)'S AND V(I)'S ARE DIRECTION I FORCE AND DISPLACEMENT COMPONENTS, RESPECTIVELY, IN THE MEMBER REFERENCE FRAME, AND, FOR I=4,5,6, THEY ARE DIRECTION I-3 MOMENTS AND ROTATIONS). TO ENSURE SYMMETRY, THE PROGRAM SETS S(J,I)= S(I,J), FOR I=1 THROUGH 6 AND J= 1 THROUGH I.

IF MTYPE=3, THE STIFFNESS MATRIX OF THIS BEAM IS THE MGROUP-TH MATRIX IN THE PREVIOUSLY-INPUT LIBRARY OF DIRECTLY-SPECIFIED MEMBER STIFFNESS MATRICES. IN THIS CASE, MGROUP IS AN INTEGER BETWEEN 1 AND N6X6 (SEE SECTION 1, CARD C, AND SECTION 6).

NSTA= 1+ THE NUMBER OF FINITE ELEMENTS INTO WHICH THE MEMBER IS SUBDIVIDED. IF NSTA.LT.2 IT IS AUTOMATICALLY SET EQUAL TO 2. A MAXIMUM SPECIFICATION IS 20. FOR LARGE STRUCTURE NSTA IS USUALLY 2. IF IPLOT.NE.0 THIS MEMBER WILL NOT BE PLOTTED IN THE SC4020 OUTPUT.

VIBRATION ANALYSIS CONTROL SECTION 9

THIS SECTION DESCRIBES THE DATA REQUIRED TO INITIATE THE VIBRATION ANALYSIS

THE FIRST CARD INDICATES THE NUMBER OF LUMPED MASSES TO BE INCLUDED. THIS DATA IS ONLY USED IN THE 1108 VERSION OF THE PROGRAM. NO LUMPED MASSES ARE ALLOWED IN THE 7094 VERSION.

LIST= NLUMP

FORMAT(I5)

THE FOLLOWING TWO CARDS ARE READ IN FOR EACH OF THE NLUMP MASSES

LIST= JT,X1,X2,X3

FORMAT(I5,3E15.8)

JT IS JOINT AT WHICH THE MASS IS LUMPED. X1,X2,AND X3 ARE THE GLOBAL POSITION COORDINATES OF THE CENTER OF THE LUMP RELATIVE TO THE JOINT, JT.

LIST= M,I11,I22,I33,I12,I13,I23

FORMAT(7E10.6)

M IS THE TOTAL MASS OF THE LUMP. I11, I22, AND I33 ARE THE MASS MOMENTS OF INERTIA OF THE LUMP ABOUT THE 3 GLOBAL AXES. I12, I13, AND I23 ARE THE MASS PRODUCTS OF INERTIA OF THE LUMP.

LIST= NOMODE,(ITEROT(I),I=1,NOMODE)

FORMAT(14I5)

NOMODE IS THE TOTAL NUMBER OF MODES DESIRED, INCLUDING RIGID BODY MODES. ITEROT(I) IS THE TOTAL NUMBER OF ITERATIONS TO BE PERFORMED ON MODE I.

THE INITIAL APPROXIMATION IS NOT CONSIDERED AN ITERATION. FOR RIGID BODY MODES LET ITEROT=0.

THE REMAINING DATA IS A SET OF POINT LOADS APPLIED TO THE STRUCTURE FOR WHICH THE CORRESPONDING DISPLACEMENT PATTERN CONSTITUTES AN INITIAL APPROXIMATION FOR A PARTICULAR MODE. DO NOT SUPPLY THIS DATA FOR RIGID BODY MODES. THE FOLLOWING ILLUSTRATES THE REQUIRED DATA FOR EACH MODE.

LIST= JOINT,(PJOINT(I),I=1,6)

FORMAT(I5,6E10.3)

JOINT= THE JOINT AT WHICH THE 6 COMPONENT LOAD PJOINT IS APPLIED
PJOINT(1)=POINT FORCE IN GLOBAL DIRECTION 1
PJOINT(2)=POINT FORCE IN GLOBAL DIRECTION 2
PJOINT(3)=POINT FORCE IN GLOBAL DIRECTION 3
PJOINT(4)=MOMENT ABOUT GLOBAL AXIS 1
PJOINT(5)=MOMENT ABOUT GLOBAL AXIS 2
PJOINT(6)=MOMENT ABOUT GLOBAL AXIS 3

A SEQUENCE OF THE ABOVE CARDS IS READ UNTIL A BLANK CARD INDICATES THAT THE LOADING FUNCTION FOR THE CURRENT MODE IS COMPLETE.

CONTINUED SOLUTION DATA SECTION 10

THE FOLLOWING DATA IS ONLY USED FOR CONTINUED SOLUTIONS AS INDICATED BY KIND.NE.0 ON THE FIRST DATA CARD. FOR THIS TYPE SOLUTION ALL REQUIRED INFORMATION FROM PREVIOUS EXECUTIONS IS READ FROM AN INPUT TAPE, ISRCH1. REQUIRED CARD INPUT FOLLOWS. THIS DATA IS READ AFTER THOSE CARDS DISCUSSED IN PARTS A. AND B. OF SECTION 1.

LIST= IPTCHG

FORMAT(I5)

THE ABOVE CARD IS NOT READ IN IF OPT(6)=0. OPT(6) IS DEFINED IN BLOCK DATA, SECTION 11.
IF IPTCHG.NE.0 CHANGES ARE TO BE MADE IN THE PLOTTING SPECIFICATIONS ACCORDING TO THE PLOTTING PARAMETERS DEFINED IN BLOCK DATA.
IF IPTCHG=0 THE PLOTS WILL BE ORIENTED AND SCALED THE SAME AS THOSE OF THE PREVIOUS EXECUTION.

LIST= NOMODE,(ITEROT(I)=1,NOMODE), SEE SECTION 9

LIST= MODEA,ITERA

FORMAT(2I5)

MODEA AND ITERA INDICATE THE INITIAL MODE AND ITERATION OF THE CONTINUED SOLUTION. IF IT IS DESIRED, THEY MAY SPECIFY A POINT PRIOR TO THE TERMINATION POINT OF THE PREVIOUS EXECUTION.

LIST= JOINT,(PJOINT(I),I=1,6), SEE SECTION 9

CONTROL PARAMETERS — SECTION 11

CERTAIN CONTROL PARAMETERS ARE DEFINED IN AN EXECUTABLE ROUTINE THAT IS CALLED FROM THE MAIN PROGRAM BEFORE ANY OTHER PROCESSING IS BEGUN.

ZERO = 0.1E-20

ZERO IS A REAL CONSTANT WHICH IS USED FOR ZERO TESTS OF FLOATING POINT NUMBERS

ANORM = 1.0E+04

ANORM IS THE FACTOR TO WHICH THE GENERALIZED MASS OF EACH MODAL APPROXIMATION IS NORMALIZED.

ISRCH1 = 11
ISRCH2 = 14
ISRCH3 = 10
ISRCH4 = 13

ISRCH1 THRU ISRCH4 ARE VARIABLE SCRATCH STORAGE UNIT ASSIGNMENTS (EITHER TAPE OR DRUM UNITS). ISRCH5 IS NOT USED

MEMPT = 1
MODDT = 3
MEMMAX = 100

```
   MEMPT IS THE SECONDARY STORAGE UNIT UPON WHICH THE MEMBER PROPERTY
        FILE IS STORED
   MODDT IS THE SECONDARY STORAGE UNIT UPON WHICH THE MODE SHAPE FILE
        IS STORED
   MEMMAX IS MAXIMUM NUMBER OF MEMBERS ALLOWED ON ANY ONE MEMBER
        PROPERTY OR MODE SHAPE FILE

   DO 10 I=1,20
10 OPT(I) = 0

   OPT(2) =-1
   OPT(3) = 1
   OPT(4) = 1
   OPT(5) = 1
   OPT(6) =-1

   THE ARRAY OPT(20) DEFINES THE OUTPUT OPTIONS OF THE PROGRAM.
   WHEN EQUAL TO ZERO THE FOLLOWING OPTIONS ARE SURPRESSED
```

OPT	VALUE	RESULTING OUTPUT
1	N	N FILES ARE WRITTEN ONTO OUTPUT TAPE, ISRCH1, EACH FILE CONTAINS THE INFORMATION REQUIRED TO CONTINUE THE SOLUTION IN A SUBSEQUENT RUN
2	1	COMPLETE DISPLACEMENT FUNCTION INFORMATION FOR EACH ITERATION OF EACH MODE
2	-1	COMPLETE DISPLACEMENT FUNCTION INFORMATION FOR THE LAST ITERATION OF EACH MODE
3	1	ITERATIVE STATIC SOLUTION INFORMATION
4	1	EIGENSOLUTION INFORMATION FOR EACH ITERATION OF EACH MODE
5	1	TIMES REQUIRED FOR VARIOUS OPERATIONS THROUGHOUT THE PROGRAM
6	1	PLOTS ARE GENERATED FOR EACH ITERATION OF EACH MODE
6	-1	PLOTS ARE GENERATED FOR THE LAST ITERATION OF EACH MODE
7	1	MODE SHAPES ARE PLOTTED WITHOUT THE ORIGINAL STRUCTURE PLOT
8-20	**	NOT USED IN THIS VERSION

```
ITMAX = 2
TOLER = 5.0E-07

ITMAX IS THE MAXIMUM NUMBER OF ITERATIONS EXECUTED PER STATIC
SOLUTION, TOLER IS THE STATIC SOLUTION CONVERGENCE CRITERION

DISP = 1.0
ROT = 0.01

THE ABOVE DATA IS USED FOR EXTERNALLY UNSTABLE STRUCTURES ONLY
DISP = MAGNITUDE OF RIGID BODY TRANSLATION
ROT = MAGNITUDE OF RIGID BODY ROTATION
THESE VALUES WILL BE AUTOMATICALLY NORMALIZED ACCORDING TO ANORM
DURING THE SOLUTION PROCESS.

NMXPG = 54
NUNIT1 = 8
NUNIT2 = 4
NUNIT3 = 3
NUNIT4 = 9
NUNIT5 = 9
NUNIT9 = 1
LREC9 = 2000
LREC4 = 1000

THE ABOVE PARAMETERS ARE USED PRIMARILY BY THE STATIC ANALYSIS
ROUTINES

NMXPG = MAXIMUM NUMBER OF LINES TO BE PRINTED PER PAGE OF OUTPUT
NUNIT1, NUNIT2, NUNIT3, NUNIT4, NUNIT5, AND NUNIT9 DEFINE
    SECONDARY STORAGE UNITS.  NOTE THAT

              NUNIT4 = NUNIT5
              NUNIT3 = MODDT
              NUNIT9 = MEMPT

LREC9 = MAXIMUM LENGTH OF THE MEMBER PROPERTY FILE
LREC4 = BASIC RECORD LENGTH LIMITATION OF THE STATIC ANALYSIS
        ROUTINES

MAR = 200
SCALE = 0.05
NVIEWS = 1
ALFA(1) = 0.0
ALFA(2) = 0.0
ALFA(3) = 0.0
IXDIR(1) = 3
IYDIR(1) = 1
IXDIR(2) = 1
```

```
IYDIR(2) = 3
IXDIR(3) = 2
IYDIR(3) = 3

THE ABOVE DATA DEFINES THE PLOTTING PARAMETERS AS FOLLOWS

   MAR = NUMBER OF RASTERS ALLOWED FOR THE MARGIN
   SCALE = SCALE FACTOR FOR MAXIMUM DISPLACEMENT.
           MAX DISP = SCALE*(MAXIMUM MEMBER LENGTH)
   ALFA(I) = ROTATION OF PLOT ABOUT GLOBAL AXIS I
   NVIEWS = NUMBER OF VIEWS PLOTTED PER MODE SHAPE
   IXDIR(I) = X COORDINATE AXIS FOR VIEW I
   IYDIR(I) = Y COORDINATE AXIS FOR VIEW I

IF THIS IS A CONTINUED SOLUTION AND THE PLOTS ARE TO BE ALTERED.
SCALE IS ASSUMED TO BE A FACTOR BY WHICH THE MAXIMUM PLOTTED
DISPLACEMENT OF THE PREVIOUS EXECUTION IS MULTIPLIED.

LUTPLN = 2

LUTPLN IS ONLY USED FOR THE L. U. T. ANALYSIS AND INDICATES IN
WHICH PLANE THE MODES ADE TO BE OBTAINED.

    LUTPLN = 1 INDICATES THE PLANE COMMON TO THE TOWER AND VEHICLE
    LUTPLN = 2 INDICATES THE PLANE NORMAL TO PLANE 1
```